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Optimization of mixed casting processes considering discrete ingot sizes †

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Abstract

Using linear programming (LP), this research devises a simple and comprehensive scheduling methodology for a complicated, yet typical, production situation in real foundries: a combination of expendable-mold casting, permanent-mold casting and automated casting for large-quantity castings. This scheduling technique to determine an optimal casting sequence is successfully applied to the most general case, in which various types of castings with different alloys and masses are simultaneously produced by dissimilar casting processes within a predetermined period. The methodology proves to generate accurate scheduling results that maximize furnace or ingot efficiency. For multi-variable and multi-constraint optimization problems per se, it provides an extremely practical solution which is readily implemented in most real-world casting plants. In addition, incorporating ingot adjustment from the reality of discrete ingot size, this LP scheduling can assist the casting industry in strengthening its competence by heightening ingot utilization as well as satisfying due dates.

Keywords: Expendable-mold and permanent-mold casting; Scheduling; Optimization; Linear programming (LP); Melting furnace; Discrete ingot size

1. Introduction

Casting, or the process of a foundry, is a very efficient and cost-effective manufacturing process which can transform raw materials into discrete products. It is also one of the earliest net shape manufacturing methods because castings are usually close to final shapes and thus require little post-processing. A useful part is manufactured by simply introducing molten metal into a mold with a cavity that has desired geometry and letting the molten metal solidify. Casting can produce intricate shapes in a single piece, ranging in various sizes. Cylinder blocks, cylinder heads, transmission cases, pistons, turbine disks, automotive wheels, various aircrafts and vessel components are some of modern castings. Castings are among the highest volume, mass-produced items manufactured by the metalworking industry. Today, the foundry industry is a large and increasingly technical industry [1, 2].

A foundryman must schedule casting sequences or establish an elaborate production plan by employing limited resources in a foundry. To improve productivity and/or minimize production time and cost, optimization of a production plan or optimal scheduling is a critical step. There have been extensive research efforts to develop a variety of optimization techniques and apply them to numerous fields in mechanical engineering including scheduling [3-9]. Also, in steel making, researchers have performed continual investigations for scheduling or production planning of a steel mill. Many of them adopted mathematical pro-

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gramming as a main tool to solve diverse scheduling problems associated with iron and steel manufacturing [10-15]. Nevertheless, the vast majority of the previous researches focused on continuous casting in a steel mill with a mass production environment. Very few studies were directed to scheduling of a foundry. On the other hand, some of them were extended to scheduling of slab, billet or bloom making, and hot and cold rolling which follow the iron and steel manufacturing process in a large-scale steel plant [16-18].

Most ordinary foundries, medium to small-sized ones, practicing various types of casting processes have a distinctive production environment, which is very different from that of a large-scale steel mill. For instance, many foundries often manage job-shop type production and start casting diverse kinds of castings upon receiving irregular orders from customers. Therefore, short-term based scheduling is needed depending on each specific order from a customer. Frequently, yet irregularly, a foundryman faces complicated scheduling problems of manufacturing various kinds of products with current manufacturing facilities in the foundry. Even worse, the scheduling problems change every time according to the orders, and they have to be solved instantly for a feasible and optimal production plan to meet the order within a due date. Hence a simple and practical scheduling methodology is vital. Unfortunately, since the sophisticated algorithms or approaches from the previous researches mentioned above are designed for large-scale iron or steel-making in a mass production environment, they cannot be practically applied to most job-shop type foundries in the real world.

There are only a handful of studies available to solve the problem of casting scheduling in a foundry [19, 20]. They use usage percentage of molten metal in casting to assess the efficiency of a scheduling solution. In a case study of an oversimplified fictitious foundry, a foundryman receives an order of 60 castings made from a single alloy in 10 shifts [19]. *[C1]*He and his coworkers rigorously apply genetic algorithm (GA) approaches for finding a solution to the maximum metal utilization. However, their GA model cannot be applied to a foundry in the real world because it is inappropriate to casting scheduling, excluding any time constraints and completely neglecting the production conditions of a real foundry such as presence of discrete ingots, multiple types of alloys, furnaces and machines, as well as the computational complexity of GA itself [21]. As a deterministic optimization methodology, linear programming (LP) is regarded as far superior to a stochastic method like GA if global optimization is guaranteed in the considered problem, because LP is simpler and easier to apply than any other stochastic method [22]. Additionally, in the formulation of casting sequence optimization, LP does not fail to include all major parameters involved in casting as constraints and to describe precisely entire casting processes commonly observed in reality. This feature will be discussed further in detail in the following sections.

This research aims to devise an LP-based optimization model for casting scheduling in most job-shop type foundries. The LP model generates an optimal casting sequence resulting in the maximum use of molten alloy. Also, an augmented scheme considering discrete ingot sizes enhances the utilization percentage further. Incorporating the amount of castings produced in each shift as the primary variable, we prove that the objective function to maximize alloy utilization percentage and entire constraints reflecting real casting conditions can be perfectly represented in linear forms. This paper presents appropriate arguments and verification in the following order. Descriptions of casting processes and scheduling in a real foundry, and rigorous mathematical formulation (LP) for casting scheduling are in Sections 2 and 3, respectively. Detailed explanation on adjustment of the quantity of ingots based on discrete ingot sizes follows in Section 4. Section 5, a numerical experiment, leads to the application of the proposed LP model to scheduling of principal casting processes, which are commonly practiced in a typical foundry. Analysis of scheduling results and ingot adjustments are also discussed in the case study. Lastly, brief comments on the summary and contribution of the research are addressed in Section 6.

2. Casting scheduling in a foundry

The processes of a foundry, or casting processes as they are often called, can be divided into expendable-mold casting process and permanent-mold casting process. Expendable-mold casting can use a mold only one time for each casting operation because the mold must be destroyed to acquire a casting after solidification of molten alloy. In permanent-mold casting, on the other hand, a mold usually made of metal or graphite is repeatedly used-thus ‘permanent’-

to cast numerous identical castings successively. To remove (or 'eject') the casting, the permanent-mold should be dismantled. To facilitate this dismantling (or 'opening') job, a permanent-mold is designed to be separated into two halves in general [1, 2, 23].

In this paper, the most prevalent casting processes in each category, sand casting and pressure casting, are selected as candidate processes to which the LP-based optimization model is applied. 'Sand-mold casting' or 'sand casting,' producing the largest quantity of castings in tonnage worldwide, is the most principal casting method in foundries. It belongs to expendable-mold casting because a sand-mold must be demolished to secure a solidified casting. Since sand casting uses a simple and inexpensive sand-mold that duplicates the shape of a casting instead of a sophisticated machine, it is a cost effective and versatile process. Castings are obtained from pre-fabricated sand-molds as shown in Fig. 1, where the number of castings equals that of sand-molds. One thing worth commenting on sand casting is that the time between successive casting productions is maintained very short compared to permanent-mold casting, which will be further discussed in detail.

On the other hand, 'pressure casting,' one of the most significant processes in permanent mold casting, requires a sophisticated and costly machine upon which a mold is installed. A casting machine incorporates a function of molten metal injection and an apparatus to engage (or clamp) and disengage two halves of a mold according to each phase in a casting cycle. Pressure casting is known to produce castings with finer microstructure, and thus has far superior mechanical properties to sand castings [1, 23, 24].

Recently, more and more foundrymen who have relatively regular demands in a large quantity from customers build an automated casting line in order to produce high quality castings at a higher rate. Under this manufacturing environment, two or more types of castings can be produced in a single automated casting line, which will be called as 'automated casting' for further discussions.

Fig. 1. Sand molds.

Suppose a foundry receives a mixed order with a due date of producing r_n for each type n casting ($n = 1,\dots,N$) made from various kinds of alloys. Then, with the current casting facilities of P furnaces of the maximum capacity $\bar{W}_{pm}$ and Q casting machines, he must decide x_{nm}, i.e., production quantity of type n casting with unit mass w_n, for each shift m ($m = 1,\dots,M$) to satisfy the exact order quantity of r_n in M shifts. Linear programming is appropriate for this scheduling problem of determining casting sequence. This is because LP can handle multi-variables and multi-constraints involved in the formulation of casting operations which have to simultaneously produce several types of castings with different alloys and masses within due dates. Also, the inherent linearity of the objective function and major constraints in casting scheduling qualifies LP as a robust tool for this complicated and real manufacturing problem.

In a typical foundry, various castings in different sizes are made out of molten alloy, which is obtained by melting ingots, e.g., solid-state raw material, in a furnace (Figs. 2 and 3) for every shift of a casting operation. Since two different kinds of alloys cannot be melted together in the same furnace, a foundryman necessitates multiple furnaces to produce simultaneously mixed products with different alloys. Nevertheless, foundries are usually equipped with only a minimum number of furnaces. In addition, the capacities of melting furnaces are often not identical because each furnace is purchased one by one, partly for

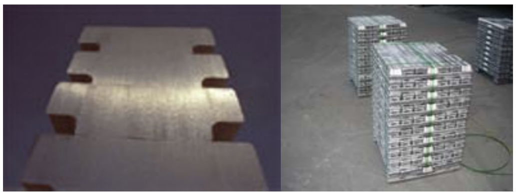

Fig. 2. Aluminum ingots.

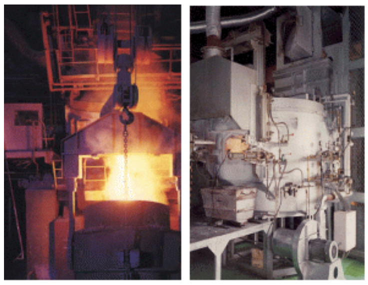

Fig. 3. Melting furnaces.

economic reasons. In each shift, molten metal is ladled from a furnace until it is used up or the planned production is achieved 100%. Production planning by personal discretion only based on intuition and experience often results in either excessive remains or shortage of molten metal. In either case, waste of raw material, energy and time or tardiness in delivery to a customer, is the painful price a foundryman must pay.

To tackle the problem of scheduling or production planning of casting, linear programming must be developed such that its objective function can measure the quality of the decision effectively. Since one of the most significant process parameters affecting manufacturing cost is molten alloy, researchers used the percentage use or average efficiency of the molten alloy as an evaluator of casting scheduling [19, 20]. At the same time, since a foundryman cannot cast more than the amount of the molten metal in a furnace, molten metal quantity induces a constraint in the LP formulation. A furnace itself is another critical constraint. A furnace used in a specific shift cannot be successively used in the next shift because it takes a significant period of time to clean and melt new ingots in the furnace. Consequently, each particular furnace is used on alternate shifts to melt an identical type of ingot. A furnace used in shift m must be idle in shift m+1, and thus at least two furnaces are necessary to melt ingots of identical alloy.

The last constraint is due to the very nature of a particular casting process. In pressure casting, a permanent mold is installed on a casting machine as described above. Thus, a foundryman must wait for a casting to solidify in the mold and be ejected out of it in order to commence the next casting. Actually, a cycle time in pressure casting is composed of mold closing, metal pouring and injection, solidification, mold opening, ejection of a casting, and mold chilling by spraying parting/cooling agent. Therefore, production by pressure casting is governed by the time constraint, which is not true in sand casting. In sand-mold casting, one does not have to wait for a casting to solidify before pouring new molten metal for the next casting as long as sufficient numbers of pre-fabricated sand-molds exist in the foundry. Therefore, in sand casting, the time elapsed between two successive castings is molten metal pouring time only, which is negligible compared to the cycle time in pressure casting. This conspicuous difference in casting operations constitutes a major distinction in the time constraint of LP formulation.

3. Linear programming

3.1 Terminology and background

Linear programming is a widely used technique of mathematical programming, and many solid applications to real-world problems are quoted in the literature [22, 25-27]. The goal is to develop a comprehensive linear model for casting scheduling problems. Refer to Nomenclature or the list of notations at the head of this paper for detailed definitions of variables and parameters employed in this paper.

The foundry of interest in this research is to cast a set of products consisting of N types. r_n items should be produced for each type n, $n = 1,\ldots,N$, which correspond to the amount of the order. Also, the entire casting operations must be completed in M shifts as requested by a customer.

In our problem setting, it is hypothesized that P furnaces, each containing a specific molten alloy of type p, $p = 1,\ldots,P$, are used concurrently in each shift m, $m = 1,\ldots, M$. P should be less than or equal to N, implying that more than one casting type can be made of single molten alloy. One important requirement concerning melting furnaces is that a previously used furnace needs at least one shift off for cleaning, recharging and melting of new ingots. Hence, at least two furnaces should be prepared for melting a type of alloy, taking turns in each shift. Since, in general, the furnaces that melt the identical alloy may have different capacities with each other, we denote by $\bar{W}_{pm}$ the maximum capacity of the furnace that melts type p alloy in the m-th shift, $p = 1,\ldots, P$, $m = 1,\ldots, M$.

Note that solid-state alloy is provided only as the form of an ingot with a discrete size, denoted by ρ_p for alloy of type p. If $\bar{W}_{pm}$ of a furnace is not exactly a multiple of ρ_p, the furnace cannot fill the molten alloy up to its maximum capacity. This feature requires another parameter, namely, W_{pm}, to represent the actual amount of molten alloy that furnace p contains in the m-th shift. Since W_{pm} must be a multiple of ρ_p, there is a positive integer ν_{pm} such that

$$W_{pm} = \nu_{pm}\,\rho_p,\ p = 1,\ldots, P,\ m = 1,\ldots, M. \quad (1)$$

Another entity that takes a part in the casting procedure is a casting machine. Not only is a casting made of a unique molten alloy, it is also cast by a unique casting machine. Suppose that Q different casting machines are working in the foundry. In common with the melting furnace, a casting machine may produce more than one casting type, e.g., as in automated casting. Moreover, sand casting requires no casting machine as mentioned before. Hence Q should be less than or equal to N.

The primary variable of the LP model is x_{nm}, the number of type n castings manufactured in the m-th shift. The objective of the casting scheduling is to determine x_{nm} for all N products and all M shifts. Except for the trivial necessity that x_{nm} must be a non-negative integer, the constraints raised by x_{nm} are sorted into three folds: the quantity of castings, mass, and time.

First, recall that r_n products should be manufactured for each casting type n during M shifts or by the end of the casting operation. Thus, the constraint on the quantity of products can be written as

$$\sum_{m=1}^{M} x_{nm} = r_n, \quad n = 1,\ldots,N \tag{2}$$

The next constraint is the mass of products. In each shift, the total mass of products that are made of a molten alloy cannot be greater than the amount of the molten alloy provided by a furnace. Denote by $r(p)$, $p = 1,.., P$, the number of product types that are manufactured out of the molten alloy in furnace p. Subsequently, denote by $p(i)$, $i = 1,..., r(p)$, indices of castings that are manufactured from furnace p. Then, the total mass of castings manufactured from furnace p in the m-th shift is $\sum_{i=1}^{r(p)} w_{p(i)} x_{p(i)m}$, where $w_{p(i)}$ is the unit mass of product type $p(i)$. Since this value should be bounded by the actual amount of molten alloy W_{pm}, the following inequality is obtained.

$$\sum_{i=1}^{r(p)} w_{p(i)} x_{p(i)m} \le W_{pm}, \quad p = 1,\ldots,P, m = 1,\ldots,M \tag{3}$$

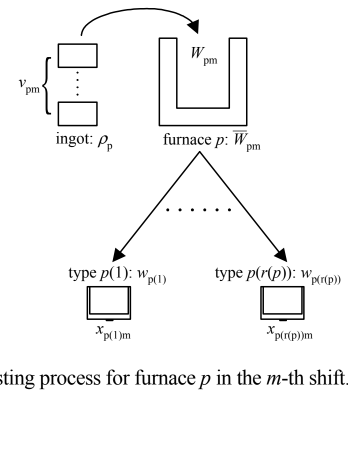

Fig. 4. Casting process for furnace p in the m-th shift.

Fig. 4 provides an illustration to assist understanding the above mass constraint and the relationship among $\overline{W}_{pm}$, W_{pm}, and ρ_p, where distribution of molten alloy from furnace p in the m-th shift is depicted.

Finally, consider the constraint on the processing time. Suppose it takes t_n, the cycle time, for a casting of type n to be manufactured. Since x_{nm} castings are cast in the m-th shift, it takes $t_n x_{nm}$ in total for x_{nm} in the m-th shift. Recall that each item of a product type is cast by its corresponding casting machine and more than one product type may share a machine. Moreover, only one item can be cast by the casting machine at a time. This implies that for a casting machine manufacturing more than one product type, the total processing time will be the sum of the time span needed for each product type. To depict this property, let us denote by $s(q)$, $q = 1,\ldots, Q$, the number of product types that are manufactured by casting machine q, and denote by $q(j)$, $j = 1,\ldots, s(q)$, the corresponding indices of product types. Since the casting operation of any machine should be completed within T_m, the available time assigned to the m-th shift, the constraint on the total processing time of casting machine q in the m-th shift is derived as

$$\sum_{j=1}^{s(q)} t_{q(j)} x_{q(j)m} \le T_m, \quad q = 1,\ldots,Q, m = 1,\ldots,M \tag{4}$$

Fig. 5 illustrates time slots for casting machine q operating in the m-th shift.

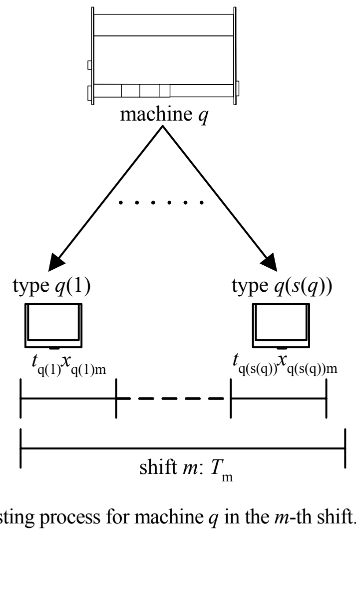

Fig. 5. Casting process for machine q in the m-th shift.

3.2 Model formulation

In this research article, a scheduler's objective is to discover all the primary variables x_{nm} which minimize the wasted amount of molten alloy or maximize the efficiency of alloy consumption. Molten alloy from a furnace is spent without being mixed with any other alloy, and one cycle of the casting campaign is launched and completed within each shift. Taking these characteristics into account, we introduce an arithmetic average efficiency of molten alloy with respect to both melting furnaces and operation shifts [19, 20].

Referring to (3), E_{pm}, the ratio of the total mass of manufactured products to the amount of molten alloy provided by furnace p in the m-th shift, is derived as

$$E_{pm} = \sum_{i=1}^{r(p)} \frac{w_{p(i)} x_{p(i)m}}{W_{pm}} \tag{5}$$

As P furnaces are used concurrently in the m-th shift, E_m, the average efficiency of the m-th shift with respect to melting furnaces, is written as

$$E_m = \frac{1}{P} \sum_{p=1}^{P} \sum_{i=1}^{r(p)} \frac{w_{p(i)} x_{p(i)m}}{W_{pm}} \tag{6}$$

Likewise, averaging again E_m for all $m = 1,\ldots, M$, i.e., for the entire shifts, our objective function E, the average efficiency of the whole casting operations is obtained as

$$E = \frac{1}{MP} \sum_{m=1}^{M} \sum_{p=1}^{P} \sum_{i=1}^{r(p)} \frac{w_{p(i)} x_{p(i)m}}{W_{pm}} \tag{7}$$

Merging three constraints (2)-(4), x_{nm} being a non-negative integer, and maximizing the objective function (7), the LP model is formulated as follows.

$$\text{Maximize: } \frac{1}{MP} \sum_{m=1}^{M} \sum_{p=1}^{P} \sum_{i=1}^{r(p)} \frac{w_{p(i)} x_{p(i)m}}{W_{pm}} \tag{8}$$

Subject to

$$\sum_{m=1}^{M} x_{nm} = r_n, \quad n = 1,\ldots,N \tag{9}$$

$$\sum_{i=1}^{r(p)} w_{p(i)} x_{p(i)m} \le W_{pm}, \quad p = 1,\ldots,P,\ m = 1,\ldots,M \tag{10}$$

$$\sum_{j=1}^{s(q)} t_{q(j)} x_{q(j)m} \le T_m, \quad q = 1,\ldots,Q,\ m = 1,\ldots,M \tag{11}$$

$$x_{nm} \ge 0,\ \ x_{nm} \in \mathrm{N},\ n = 1,\ldots, N,\ m = 1,\ldots, M \tag{12}$$

Eqs. (9)-(12) mean that the above LP model has N equality constraints and $(P+Q+N)M$ inequality constraints. Also, the constraint (12) lets the proposed LP model belong to integer programming (IP). By applying the model to entire typical casting environments and testing its effectiveness, this study proves its robustness in various dissimilar casting conditions commonly met in the real world on a daily basis.

One significant advantage of this model is that no major modification is needed in the formulation to express a variety of realistic casting situations. For instance, if the foundry under consideration practices only sand casting where the cycle time is negligible, we can describe such a foundry by setting t_n =0 for all $n = 1,\ldots, N$, i.e., the time constraint (11) is removed.

Assume, on the other hand, that all the products are made by low-pressure casting only. The number of casting machines Q is equal to that of product types N, i.e., each product type is made by the machine that is assigned exclusively to the product type. Then, $s(q) = 1$, $\forall q = 1,\ldots, Q$, and the time constraint (11) becomes fully decoupled with respect to product type n.

As the most general case, a foundry may concurrently practice three kinds of casting processes together–sand casting, low-pressure casting, and automated casting by which multiple types of products can be made on an automated casting line. We can easily describe this complicated case by setting $s(q) \ge 2$ for at least one q and assigning the cycle time such that $t_n = 0$ for sand casting, and $t_n > 0$ for low-pressure casting and automated casting.

4. Ingot adjustment

We now turn to another main contribution of this study, namely how to raise the efficiency of casting process by adjusting the number of ingots charged in the furnace. This technique is motivated by the characteristics of the casting operation that solid-state raw material in the pre-melting phase is provided only as a set of solid ingots with discrete size. Thus, it restricts the amount of molten alloy in a melting furnace to be a multiple of the unit ingot mass. Another fact to remind is that if there is a residue or remains of molten alloy at the end of a shift, it cannot be recycled in the next shift. Therefore, the best way to raise the average efficiency of the casting operation is that *after* obtaining the scheduling result from the LP model, we reduce the number of charged ingots that would be wasted otherwise *before* actually executing the casting sequence.

The necessary condition for adjusting the number of charged ingots is that in a shift, the difference between the amount of molten alloy in a furnace and the total mass of the products scheduled is greater than or equal to the unit mass of an ingot. To exemplify this condition, assume that the following inequality holds true for melting furnace p in the m-th shift.

$$W_{pm} - \sum_{i=1}^{r(p)} w_{p(i)} x_{p(i)m} \geq \rho_p \ , \tag{13}$$

where $r(p)$ and $p(i)$ are as defined above (see Fig. 4). After confirming this relation, a foundryman using our LP method can eliminate a priori redundant ingots. Remind that W_{pm} is a multiple of ρ_p by factor ν_{pm} in Eq. (1). Instead of ν_{pm}, we can find an adjusted number of ingots, namely $\tilde{v}_{pm}$, using the following equation:

$$\tilde{v}_{pm} = \min_k \left\{ k \in \mathbf{N} \mid k\rho_p - \sum_{i=1}^{r(p)} w_{p(i)} x_{p(i)m} \geq 0 \right\} \tag{14}$$

$\tilde{v}_{pm}$ is the minimum number of ingots that are necessary to produce the schedule $x_{p(1)m}, x_{p(2)m}, \ldots, x_{p(r(p))m}$ assigned to furnace p in the m-th shift. Consequently, the amount of molten alloy from furnace p can be adjusted to

$$\tilde{W}_{pm} = \tilde{v}_{pm} \rho_p \tag{15}$$

It is evident that $\tilde{W}_{pm} \leq W_{pm}$ from Eqs. (1) and (13)-(15). By substituting W_{pm} with $\tilde{W}_{pm}$ in the objective function (8), an improvement in the average efficiency is guaranteed always.

Note that the proposed adjustment scheme does not belong to the optimization procedure; it should be performed only after the completion of LP because it requires the scheduling result x_{nm}. Adjusting ingot numbers before calculating LP is hardly applicable, if not impossible, for the following reasons. First, it can shrink the range of feasible solutions to a significant extent. The total mass of products to be cast in the entire shifts is $\sum_{m=1}^{M} \sum_{n=1}^{N} w_n x_{nm}$. In intuitive terms, a superior policy might be to set up the number of ingots as barely enough as to produce the amount $\sum_{m=1}^{M} \sum_{n=1}^{N} w_n x_{nm}$ before calculating the LP model. However, according to our numerical experiments (of which results are omitted for space limitation), the proposed LP could not find a feasible solution for many case studies, including the one that will come in the next section. Second, for a foundryman, whether adjusting the number of ingots prior to performing LP or after LP completion is not much of an interest as long as it is guaranteed that the minimum number of ingots is actually melted in his furnace. An identical number of ingots will be obtained by either way if a priori ingot elimination before LP maintains the same range of feasible solutions.

5. Numerical experiment

5.1 Problem setting

In this section, a test problem is used to evaluate the computational characteristics of the proposed LP model. The LP model is implemented with Premium Solver Platform® v8.0 [28], a standard linear programming software package. The parameters of the test problem are data taken from a foundry [29].

Consider a foundry employing two types of furnaces (P = 2), which are assigned to each alloy type: one melting A383 alloy (p = 1) and the other ASTM B26 356-T6 alloy (p = 2), respectively. One of the furnaces is used in every odd shift while the other at rest for cleaning and melting of new ingots, and vice versa in even shifts. As a result, four furnaces exist in the foundry. The foundryman must complete casting operations in 12 shifts, i.e., M = 12. Indices and parameters of the furnaces are summarized in Table 1. Since the maximum capacity $\bar{W}_{pm}$ is a multiple of the unit ingot mass ρ_p for both p = 1 and 2, the amount of provided molten alloy W_{pm} is identical to $\bar{W}_{pm}$ for each p and m.

This foundry is to cast four types of products denoted by A, B, C and D (N = 4): 'A' for a cylinder block (n = 1), 'B' for an automatic transmission case (n = 2), 'C' for an axle housing (n = 3) for heavy construction machinery, and 'D' is a cylinder block for heavy-duty vehicle (n = 4). Product type indices and other parameters are summarized in Table 2. As shown in the table, castings B and C are made of the alloy from furnace p = 1. Castings A and D are made of the alloy from furnace p = 2.

The casting process in the foundry is laid out as the most general, i.e., it consists of major casting

Table 1. Specification of alloy and melting furnace.

Molten alloy	A390		ASTM B26 356-T6	
Furnace index ($P=2$)	$p=1$		$p=2$	
Unit ingot mass ρ_p (kg)	100		150	
Max. capacity $\overline{W}_{pm}$ (kg)	2,500	2,300	3,000	2,700
Amount of alloy W_{pm} (kg)	2,500	2,300	3,000	2,700
Factor ν_{pm}	25	23	20	18
Assigned shift ($M=12$)	odd	even	odd	even

Table 2. Product and machine specification.

Product	A	B	C	D
Type index ($N=4$)	$n=1$	$n=2$	$n=3$	$n=4$
Number of products r_n	350	300	200	180
Mass w_n (kg)	52.2	48.6	65.0	80.0
Furnace index (p)	2	1	1	2
Cycle time t_n (min.)	13.6	12.5	18.1	0
Machine index ($Q=2$)	1	1	2	–

processes–sand casting, low-pressure casting, and automated casting. As displayed in Table 2, product D is manufactured by sand casting. Thus, the cycle time t_4 is set to be 0 and no casting machine is assigned. Since product C is cast by low-pressure casting, a casting machine ($q=2$) for the product is allotted with the cycle time (t_3 in Table 2) of 18.1 minutes, which is sampled from a foundry [29]. Finally, products A and B which have large production quantities (350 and 300, respectively) are manufactured in an automated casting line ($q=1$) with cycle times of 13.6 and 12.5 minutes, respectively. T_m, the available time of each shift m, is 12 hours or

$$T_m = 720 \text{ (min.)}, \forall m = 1,\ldots,12 \tag{16}$$

In fact, one cycle of low-pressure casting consists of many successive phases in the sequence of mold locking, ladling, injection, solidification and cooling, mold opening, ejection of a casting, and mold spraying. Among all of them, solidification and cooling spends the longest time in one cycle. In low-pressure casting, solidification and cooling time is a complex nonlinear function of volume-to-surface-area ratio of a casting, alloy, mold, latent heat, superheat, heat transfer, etc. [30].

To calculate the solidification and cooling time of axle housing of 65 kg in mass, some assumptions are unavoidable. First, assuming the geometry of the casting being a sphere, the volume is 24.074×10^{-3} m^3 and surface area is about 0.4 m^2. The total solidification and cooling time in permanent-mold casting is calculated as follows [30]:

$$t_{total} = \frac{\rho V}{hA(T_m - T_o)}\{H + c(T_1 - T_2)\} \tag{17}$$

where ρ : density of alloy [kg/ m^3], V: volume of a casting [m^3], H: heat of fusion [cal/kg], c: specific heat [cal/kg ^{o}C], h: heat transfer coefficient [cal/kg ^{o}C sec], A: surface area [m^2], T_m: temperature of molten metal, T_o: mold temperature, T_1, T_2: initial and final temperatures [30, 31]. With the consideration of the superheat in usual casting and eutectic temperature of aluminum alloy, the holding temperature in the furnace is assumed to be 700 ^{o}C and the temperature of a solidified casting upon ejection is about 200^{o}C. Then, the total heat dissipated from the casting is about 29.357×10^6 J. Assuming again that the heat transfer coefficient of molten metal to the mold is roughly 200 W/m^2 ^{o}C and the mold temperature is 225^{o}C [32], we can calculate the heat flux of molten metal at 700^{o}C to be about 95×10^3 W/m^2. Consequently, the solidification and cooling time is approximately 766 seconds or 12.8 minutes, which is of course less than the cycle time of 18.1 minutes.

Certain discrepancy between the calculated value and real one is expected due to several factors. For example, the axle housing is not a sphere but a very complex shape. Accordingly, the surface area is not equal to 0.4 m^2, which inevitably causes some inaccuracy. Unfortunately, the exact surface area is not available. In addition, the heat flux of the casting does not remain constant, but is a complicated nonlinear function of time. It requires a sophisticated numerical calculation to obtain an accurate result, which is far beyond the scope of this research.

5.2 Scheduling result

The results of the numerical experiment are divided into two phases: the scheduling result before and after ingot adjustment. The optimal solution calculated from the proposed LP model before ingot adjustment is displayed in Table 3. Except for a few shifts, all the efficiencies of the provided molten alloy E_{pm} are over 90%, from which a high average efficiency E = 95.81% is derived. Thus, our solution can be said to obtain the optimality in terms of the usage of molten alloy.

All the time constraints (11) are also satisfied in the solution. For example, t_3 = 18.1 (min.) for product C (n = 3) from Table 2 and T_m = 720 (min.), $\forall m$, as defined in (16). Applying (11), we obtain the maximum value of x_{3m} as 720/18.1 = 39 (integer). It is evident that all the values of x_{3m} in Table 3 are bounded by 39. In fact, the largest amount of production (x_{33}) is 38 in the shift 3. On the other hand, product D (n = 4), which is manufactured by sand casting, has no bound determined by the time constraint. Nevertheless, as is evident in Table 3, x_{4m} cannot be increased arbitrarily because it is definitely associated with x_{1m} in terms of the mass constraint (recall that products A (n = 1) and D (n = 4) share the identical alloy from furnace p = 2).

x_{1m} and x_{2m} constitute the scheduling result for automated casting. Though products A and B are cast by the identical machine, the production quantities are not evenly divided between A and B in each shift. Instead, their amounts in a shift are much inclined to either one of two product types. This consequence is due to the fact that product A shares the same furnace (furnace p = 2) with product D, as mentioned, while product B does with product C (furnace p = 1). In other words, products A and B are associated with each other in terms of the time constraint (11), but they are associated with another product, i.e., A with D and B with C, in terms of the mass constraint (10). We can easily check that all the other interconnections between the primary variables are well conceived in the final scheduling solution.

Table 3. Scheduling result before ingot adjustment.

Shift (m)	x_{1m}	x_{2m}	x_{3m}	x_{4m}	E_{1m} (%)	E_{2m} (%)
1	52	1	31	0	82.54	90.48
2	50	3	33	1	99.60	99.63
3	52	0	38	0	98.80	90.48
4	50	3	33	1	99.60	99.63
5	50	3	29	0	81.23	87.00
6	7	47	0	29	99.31	99.46
7	11	45	3	27	95.28	91.14
8	50	3	33	1	99.60	99.63
9	6	51	0	32	99.14	95.77
10	8	47	0	28	99.31	98.43
11	6	50	0	33	97.20	98.44
12	8	47	0	28	99.31	98.43
E_p (%) (p = 1,2)					95.91	95.71
Avg. E					95.81	

5.3 Procedure of ingot adjustment

After obtaining the optimal solution, the next step is to adjust the number of ingots to be charged before actually handing the scheduling result to a foundryman.

Fig. 6 illustrates the amount of molten alloy and the total mass of the scheduled products in each shift, where a shaded segment denotes the difference between the two quantities. The adjusted ingot number is determined by the ingot mass. Since ρ_1 =100 kg for furnace 1, we look for the shaded segment larger than 100 kg or unit ingot mass (Eq. (13)). Referring to Fig. 6, the shifts with such a difference are found to be shifts 1, 5 and 7. For shifts 1 and 5, the difference is in between 4 and 5 unit ingot masses, respectively, and for shift 7, it is in between 1 and 2 unit ingot masses. Therefore, 9 ingots in total or raw material of 900 kg can be saved plus energy to melt the extra

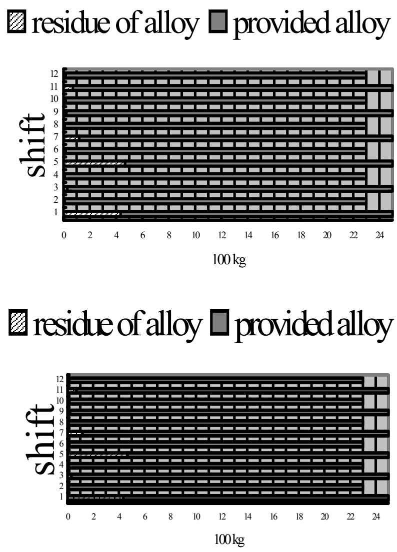

Fig. 6. Difference between the amount of molten alloy and the total mass of the scheduled products in each shift for furnace 1 and furnace 2.

Table 4. Ingot adjustment.

Shift (m)	$v_{1m} - \tilde{v}_{1m}$	$v_{2m} - \tilde{v}_{2m}$	E_{1m} (%)	E_{2m} (%)
1	4	1	98.27	95.24
2	0	0	99.60	99.63
3	0	1	98.80	95.24
4	0	0	99.60	99.63
5	4	2	96.70	96.67
6	0	0	99.31	99.46
7	1	1	99.25	95.94
8	0	0	99.60	99.63
9	0	0	99.14	95.77
10	0	0	99.31	98.43
11	0	0	97.20	98.44
12	0	0	99.31	98.43
E_p(%) (p=1,2)			98.84	97.71
Avg. E			98.28	

ingot. Likewise, from Fig. 6 we can also find those shifts wherein the number of ingots charged can be reduced for furnace 2-shifts 1, 3, 5 and 7. The difference is in between 2 and 3 unit ingot masses especially for shift 5, and it is in between 1 and 2 for other shifts considering ρ_2=150 kg. Therefore, ingots or raw material of 750 kg can be saved in total.

Table 4 is the final result of our scheme where the number of ingots is adjusted for each furnace and shift. $\tilde{v}_{pm}$, the adjusted number of charged ingots, is derived from Eq. (14) and Fig. 6, and applied to Eq. (15) for obtaining $\tilde{W}_{pm}$, the amount of molten alloy from furnace p that will be actually used in the casting. Substituting W_{pm} with $\tilde{W}_{pm}$, all the efficiencies E_{pm}, E_p and E are re-calculated. Note that the augmented average efficiency is E=98.28% in Table 4, i.e., 2.47% increase compared to the previous result. This improvement demonstrates that the proposed scheme of ingot adjustment can enhance the efficiency without compromising the optimality of the scheduling result.

6. Conclusions

Scheduling theory has the potential to play an important role for improving the productivity of casting processes. This paper has reported research in which linear programming was applied to determine the optimal scheduling sequence, which maximizes the average efficiency of provided molten alloys. For this purpose, a variety of casting activities are classified in detail according to their characteristics. Also, with the numbers of products to be cast in each shift as the primary variables, the objective function and constraints on product numbers, mass, and time are derived in the framework of linear programming. In particular, motivated by the unique property of the casting process that solid-state material is provided as ingots with discrete size, the authors have proposed a scheme of adjusting the number of charged ingots to enhance the efficiency of the scheduling result. From the numerical experiment, it is well proved that the proposed LP model and the adjustment scheme can be applied to real casting environments.

The authors hope this study can contribute to initiating researches on scheduling of casting processes in many real foundries adopting job-shop style production.

Nomenclature

- N : Total number of casting types in an order
- M : Total number of shifts (production run)
- P : Total number of furnaces ($P \le N$)
- Q : Total number of casting machines ($Q \le N$)
- T_m : Amount of time available in the m-th shift, m = 1,…, M
- W_{pm} : Amount of molten alloy from furnace p in the m-th shift, p = 1,…, P, m = 1,…, M
- $\bar{W}_{pm}$: Maximum capacity of furnace p used in the m-th shift, p = 1,…, P, m = 1,…, M
- $\tilde{W}_{pm}$: Adjusted amount of molten alloy from furnace p in the m-th shift, p = 1,…, P, m = 1,..., M
- x_{nm} : Number of casting n manufactured in the m-th shift, n = 1,…, N, m = 1,…, M
- r_n : Total number of casting n that must be manufactured in M shifts, n = 1,…, N
- w_n : Unit mass of casting n, n = 1,…, N
- t_n : Cycle time of making casting n, n = 1,…, N
- ρ_p : Unit mass of an ingot charged in furnace p, p = 1,…, P
- v_{pm} : Number of ingots pre-assigned to furnace p in the m-th shift, p = 1,…, P, m = 1,…, M
- $\tilde{v}_{pm}$: Adjusted number of ingots for furnace p in the m-th shift, p = 1,…, P, m = 1,…, M
- $r(p)$: Number of casting types manufactured from furnace p, p = 1,…, P
- $p(i)$: Indices of casting manufactured from furnace p,

$i = 1,\ldots, r(p)$

$s(q)$: Number of casting types manufactured by machine q, $q = 1,\ldots, Q$

$q(j)$: Indices of casting manufactured by machine q, $j = 1,\ldots, s(q)$

E_{pm} : Ratio of the total mass of manufactured products to the amount of molten alloy provided by furnace p in the m-th shift, $p = 1,\ldots, P$, $m = 1,\ldots, M$

E_m : Average efficiency of the m-th shift with respect to furnaces, $m = 1,\ldots, M$

E : Average efficiency of the whole casting operations

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